

Differences in the Effects of Problem-Based Learning and Inquiry-Based Learning Models on Students' Mathematical Generalization Ability in Terms of Learning Motivation

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Abstract

The purpose of this study was to determine how students' mathematics generalization skills varied based on their motivation when using the Problem-Based Learning (PBL) and Inquiry-Based Learning (IBL) learning models. This study uses a quasi-experimental design using a pretest-posttest non-equivalent group design and a quantitative methodology. Students in the eighth grade at a junior high school in Banten, Indonesia during the 2025–2026 school year make up the population of this study. Purposive sampling was used to select two classes of eighth-grade pupils for the sample. Mathematical generalization ability tests and learning motivation questionnaires are used to collect data, which is then analyzed using two-way ANOVA and the Scheffe test at a significance level of 5%. The conclusion is that PBL and IBL students have the same capacity for mathematical generalization. However, students' capacity for mathematical generalization varies according on their degree of motivation. Students with low motivation differ significantly from those with high and moderate motivation in their capacity to generalize mathematical concepts, according to the results of the Scheffe test. Nonetheless, there is no discernible difference between pupils with high and moderate motivation in terms of their capacity for mathematical generalization. Learning models and learning motivation in relation to mathematical generalization abilities also interact. Therefore, it may be said that the degree of student motivation determines how the learning models affect their capacity for mathematical generalization. Therefore, teachers should consider the motivation of students in choosing learning models of mathematics.

Keywords: Problem-Based Learning, Inquiry-Based Learning, Mathematical Generalization Ability, Learning Motivation.

INTRODUCTION

Higher Order Thinking Skills (HOTS) has been seen as one of the critical competencies that need to be developed within the framework of mathematics education for the 21st century. With regards to the development of education for the 21st century, the design of mathematics education needs to take into account that students should be encouraged to think critically, solve problems, and reason mathematically using student-centered learning approaches (Mustafa, 2023). The significance of developing HOTS in the context of mathematics education has also been discussed by Pujiastuti & Haryadi (2023), according to whom, HOTS is one of the most crucial competencies that can help students understand the concepts, analyze the problems, and find out alternative solutions to mathematical problems.

Among the higher order thinking skills which have a crucial role in mathematics education, there is mathematical generalization ability. This skill allows students to see patterns, find regularities, make conjectures, and construct general rules to tackle different math problems. As stated by Radford (2021), mathematical generalization is an intellectual

operation through which students can go from particular cases to abstract rules. Moreover, according to Stephens et al. (2021), the development of generalization skills has an important part in fostering the algebraic thinking and mathematical reasoning of students.

Mathematical generalization is at the heart of mathematical activities since most mathematical ideas, theories, and theorems are established through the identification of patterns and constructing general rules. Mathematical generalization ability is significant for students to understand interconceptual relations and move their learning of math from using formulas to conceptual comprehension of mathematics, as explained by Alzahrani & Alfadhli (2023). Indeed, Suryowati & Trisianti (2022) note that mathematical generalization involves the connection between concrete experience and abstraction to develop mathematical knowledge.

The importance of the development of mathematical generalization ability has also been widely discussed in international literature. It is considered one of the cornerstones in terms of the development of mathematical reasoning since it gives opportunities for the identification of patterns, establishing connections between concepts, and creating general rules to be used in various mathematical contexts. According to García Fajardo et al. (2025), the development of mathematical generalization ability leads to the formation of algebraic reasoning and problem-solving skills. Also, according to Ellis et al. (2022), the ability to generalize is one of the main signs of deep conceptual understanding and transferring mathematical knowledge to other contexts. Thus, the development of mathematical generalization ability has become the major aim in modern mathematics education.

The role of mathematical generalization ability has been widely studied in international literature. Devi & Amir (2021) have proved that students who have developed mathematical generalization ability can easily solve nonroutine tasks and establish connections between different mathematical concepts. Naraswari et al. (2023) also proved that mathematical generalization ability affects the development of the ability to create symbolic representations and establish connections between concepts. Nonetheless, many researchers claim that students have poor mathematical generalization ability. For example, Maula et al. (2024) proved that most students experience problems with pattern recognition, the accurate use of mathematical symbols, and creating general rules based on the information received. Mathematical generalization ability allows students to find patterns, establish connections between concepts, and create general rules applicable in different situations (García Fajardo et al., 2025; Ellis et al., 2022).

These results are supported by studies of Dani et al. (2017), who proved that students' mathematical generalization ability has not yet been fully developed. The authors found out that students have problems in their ability to represent mathematical patterns and form general ideas about particular situations. In the study carried out by Muslimah et al. (2025) with regard to junior high school students of the Mathematics Teachers' Association (MGMP) of Cilegon City, students' mathematical generalization ability was found to be in need of improvement especially in the aspects of expressing regularities, representing mathematical relations, and creating symbolic generalizations. The above results indicate that the development of students' mathematical generalization ability has always been a pressing issue in junior high school mathematics education.

Low mathematical generalization ability of students is directly linked to learning processes that are still information oriented. Teacher-centered learning causes students to receive concepts directly without adequate opportunities to explore patterns, build conjectures, and discover concepts independently. Consequently, students become more accustomed to memorizing formulas rather than understanding how mathematical concepts are formed. Therefore, alternative learning models are needed to actively engage students in searching for and constructing knowledge.

Problem-Based Learning (PBL) is one learning approach thought to be capable of fostering mathematical generalization skills. This method places a strong emphasis on using contextual problems as the starting point for learning, where students are asked to research, debate, create techniques for solving difficulties, and consider their conclusions. The activities of problem analysis, self-directed learning, teamwork, problem-solving, and reflection are all part of problem-based learning, which can aid in the development of students' higher-order thinking abilities (Yu & Zin, 2023). Research by Hadi et al. (2025) and Saputri & Purwasih (2024) demonstrated the efficacy of using problem-based learning to promote higher-order thinking skills. They demonstrated how problem-based learning significantly affects students' critical thinking and a number of mathematical abilities. Yew and Goh's research (2021) demonstrates the efficacy of problem-based learning. They claim that problem-based learning enhances students' analytical abilities, knowledge transfer, and critical thinking.

It has been demonstrated that problem-based learning is an effective way to help students acquire higher-order thinking abilities, such as mathematical reasoning, problem-solving, and conceptual skills. The development of more adaptable and profound thinking techniques is encouraged in a problem-centered learning environment (Hadi et al., 2025; Simanjuntak et al., 2021). Several international studies also proved the efficiency of Problem-

Based Learning for the development of various higher-order thinking skills in mathematics. For example, according to Rafiq et al. (2023), consistent application of PBL has a positive effect on students' reasoning, problem-solving, and conceptual skills. Similar findings were made by Bron and Prudente (2024): the implementation of Problem-Based Learning led to the growth of mathematical creativity and higher-order thinking skills. Characteristics of PBL allowing learners to work on authentic problems provide them with opportunities to develop more flexible and profound thinking strategies.

Other than PBL, there is another instruction model that can also be used to foster mathematical generalization. It is Inquiry-Based Learning (IBL). Inquiry-Based Learning model focuses on investigative activities through observation, asking questions, gathering information, testing out hypothesis, and making conclusions. In such a process, learners will get an opportunity to independently discover mathematical concepts based on their learning experiences. Yuni & Fisa (2020) found that inquiry-based learning could increase learners' ability to build mathematical relationships and make generalizations based on the patterns discovered. Inquiry-Based Learning is thus expected to aid learners in optimizing their mathematical generalization abilities. Inquiry-Based Learning gives learners an opportunity to undertake systematic investigations, in which case they get a chance to construct mathematical concepts by exploring and discovering. Such a process has been proven to help in fostering learners' mathematical reasoning and argumentation skills (Doz et al., 2025; García Fajardo et al., 2025).

Inquiry Based Learning has also been found to be an effective method for improving students' mathematical thinking skills. According to Doz et al. (2025), inquiry-based learning increases learners' mathematical reasoning since they get actively involved in investigation and concept building process. Moreover, Arifin et al. (2025), in a systematic review, found that IBL positively impacts students' conceptual understanding, mathematical argumentation skills, and critical thinking skills at various educational levels. As shown by results from Sen et al. (2021), inquiry learning could improve learners' ability to conjecture, investigate mathematically, and reason. These are all vital aspects of mathematical generalization.

There are several elements that could influence the success of implementing learning models. Those elements include external factors such as learning methods and internal factors of students themselves. Learning motivation is one important element among many others for internal factors. Learning motivation refers to the motivation that triggers students to get involved in the process of learning so that learning objectives can be accomplished. According to Ryan & Deci (2020), strong learning motivation would increase students'

involvement in the learning process, hence better academic performance (Mustafa, 2024). Recent research also indicates that learning motivation contributes to students' self-regulation skills in learning mathematics. Highly motivated students tend to be better at setting learning goals, monitoring their learning progress, and maintaining effort when facing challenging tasks (Howard et al., 2021; Schukajlow et al., 2023).

Studies by Lo et al. (2022) show that students with high learning motivation tend to exhibit better persistence, participation, and academic achievement compared to those with low learning motivation. International research demonstrates that learning motivation is strongly correlated with student engagement, academic persistence, and mathematics achievement. Highly motivated students tend to show better engagement during the learning process and achieve more optimal learning outcomes (Xiao & Sun, 2021).

For the last several years, international literature has revealed that learning motivation is connected strongly to student engagement and learning achievements. As was mentioned by Nagy et al. (2022), highly motivated learners can demonstrate better engagement in the learning process. In turn, Hidayatullah et al. (2024) revealed that learning engagement serves as a mediator between motivation and mathematics achievement. It is possible to see that learning motivation not only affects learning outcomes but also the quality of involvement in the learning process. Many empirical data are available concerning the effectiveness of Problem-Based Learning and Inquiry-Based Learning approaches in developing the critical thinking, problem solving, reasoning, and concepts of mathematics. At the same time, mathematical generalization is one of the least-studied topics. Moreover, no comparative analysis of Problem-Based Learning and Inquiry-Based Learning regarding students' mathematical generalization ability exists. Nevertheless, learning motivation is an internal factor that significantly impacts student engagement in the learning process and learning outcomes.

Given this context, it is evident that there is a study gap concerning students' capacity for mathematical generalization in Problem-Based Learning and Inquiry-Based Learning with regard to learning motivation. This study is interesting in that it shows how students who study using problem-based learning and inquiry-based learning differ in their capacity for mathematical generalization while taking learning motivation into consideration as a factor that influences the learning process. It is anticipated that the current study's findings will potentially advance mathematics education and give educators a chance to select the best learning models to help pupils enhance their mathematical generalization abilities. Therefore, the purpose of this study is to examine how junior high school students' mathematics

generalization skills differ between problem-based learning and inquiry-based learning in terms of learning motivation.

METHOD

In this study, a quasi-experimental approach with pretest-posttest nonequivalent group design was utilized. This kind of design was applied for making comparison of students' mathematical generalization ability between the two groups with varied treatment of instruction.

The research process took place during the second semester of academic year 2025/2026 a junior high school in Banten, Indonesia. The population of the research consisted of all eighth grade students who were distributed among several classes with the same level of competence. The sample was determined with a purposeful sampling method considering equivalent academic abilities of the subjects as well as suggestions of the mathematics subject teachers. There were two classes involved in the research process.

The research instruments consisted of a mathematical generalization ability test and a learning motivation questionnaire. The mathematical generalization ability test comprised 5 essay questions designed based on indicators of mathematical generalization ability, namely: (1) identifying patterns or regularities; (2) expressing mathematical relationships verbally, symbolically, or visually; (3) formulating general rules based on the discovered patterns; and (4) applying general rules to different situations. The test instrument underwent expert validation and piloting to determine its validity, reliability, difficulty level, and item discrimination.

Motivation of learning questionnaire was developed consisting of 18 items derived from the indicators of learning motivation such as persisting in learning, interest in instruction, desire to succeed, learning independence, and resilience to overcome difficulties. From the results of the questionnaire, the students were classified into high, medium, and low groups of learning motivation. This instrument was verified by the experts and underwent reliability test before it is applied in this research.

Research data were collected through pretest before treatment and post-test after treatment on both sample classes. Also, the learning motivation questionnaire was provided to measure the motivation level of each student. In the instructional activity, the implementation of the PBL model took place through the stages of problem orientation, organizing students for study, investigating, developing and presenting artifacts, and evaluating the problem-solving process. Simultaneously, the implementation of the IBL model took place through the

stages of orientation, problem formulation, generating hypotheses, collecting data, testing hypotheses, and concluding.

Data analysis were processed gradually. Descriptive statistics were used to describe students' mathematical generalization ability in terms of mean, standard deviation, minimum value, and maximum value. Before the test of hypothesis, the normalcy test and homogeneity test were conducted through the Shapiro-Wilk test and Levene's test respectively, to ensure the fulfillment of assumption of parametric test. Then, the test of hypothesis was conducted through two-way ANOVA to examine the effect of learning models, learning motivation, and interaction of learning models and learning motivation on the students' mathematical generalization ability. If there was an effect on the motivation variable, Scheffe's test would be conducted to see which pairs of learning motivation groups showed significant differences. All data analyses were conducted through IBM SPSS Statistics software with 5% level of significance.

RESULTS AND DISCUSSION

The purpose of this research is to conduct the differences in generalization ability in mathematics of students who have been taught by PBL method and IBL method. This research was conducted by analyzing the data descriptively and inferentially using Two Way ANOVA.

Descriptive Statistics of Mathematical Generalization Ability

Prior to the hypothesis testing, descriptive statistics analysis is done to give a brief analysis on students' ability in mathematical generalization in both the Problem Based Learning (PBL) and Inquiry Based Learning (IBL). The descriptive analysis involves the use of the pretest and posttest scores to show whether the students have improved in mathematical generalization. Descriptive statistics findings are shown in Table 1.

Table 1. Descriptive Statistics of Students' Mathematical Generalization Ability

Learning Model	N	Pretest Mean	Posttest Mean
Problem Based Learning (PBL)	35	26.05	77.52
Inquiry Based Learning (IBL)	35	24.67	79.24

From Table 1, it can be observed that the treatment has brought about an improvement in terms of mathematical generalization ability in both the groups. The posttest mean shows that students who learned under both the PBL and IBL models have reached a moderately improved category regarding mathematical generalization ability. Descriptively speaking, it can be said that mathematical generalization ability of IBL group was slightly higher than that of PBL group.

Before conducting hypothesis test, the normality and homogeneity test were conducted on the data collected. It has been seen from the test results that the data were normally distributed and homogeneous, which signifies that the data satisfy the prerequisite to conduct the two-way ANOVA. In order to determine if there are any differences in the mathematical generalization ability of the students based on the learning models and learning motivation and interaction of two variables, two-way ANOVA test was conducted. The results of two-way ANOVA test are presented in Table 2.

Table 2. Results of Two-Way ANOVA

Mathematical Generalization Ability	Source of Variation	<i>Sum of Square</i>	<i>df</i>	<i>Mean Square</i>	<i>F</i>	<i>Sig.</i>	Interpretation
<i>Posttest</i>	Model	9,170	1	9,170	0,082	0,775	Not significant
	Motivation	1644,908	2	822,454	7,389	0,010	Significant
	Model x motivation	1126,529	2	563,264	5,060	0,009	Significant
	Error	7123,607	64	111,306			

From Table 2 shown above, it is evident that the significance level of the learning models variable is $0.775 > 0.05$. This implies that there is no significant difference in mathematical generalization skills between those students who were subjected to problem-based learning models and inquiry-based learning models. On the other hand, the significance level of the learning motivations variable is $0.010 < 0.05$, and this implies that there is a significant difference in mathematical generalization skills based on learning motivations of the students. Lastly, from the two-way ANOVA results above, it is clear that the significance level of the interaction between learning models and learning motivations is $0.009 < 0.05$.

The Interaction Between Learning Models and Learning Motivation

The results obtained through two-way ANOVA indicate that there is a significant interaction effect of the learning models and learning motivation of the students in mathematical generalization ability. This implies that the effect of the learning model on mathematical generalization ability is variable based on different levels of students' motivation. This means that the efficiency of the learning approach in enhancing mathematical generalization ability of the students is dependent on the students' motivation.

The non-significant difference between PBL and IBL can be explained by the fact that both learning models are based on similar theories. Both models are founded on constructivism and involve student-centered classroom settings. The models offer equal chances for the students to build their knowledge by investigating and reflecting on different aspects, thereby developing relatively similar higher-order thinking skills (Rafiq et al., 2022; Bron & Prudente, 2024). From the constructivist point of view, both models allow students to

explore, understand information, link mathematical ideas, and discover patterns resulting in mathematical generalization. Nonetheless, the capability of the students to take advantage of these learning activities is greatly influenced by their internal motivation.

Interaction of Learning Models and Learning Motivation on Mathematical Generalization Ability

For the purpose of getting a better understanding of the interaction effect between the two independent variables on the dependent variable, the mean scores of the post-test for each group combination were analyzed. The findings of this analysis are depicted in Table 3.

Table 3. Mean Scores of Mathematical Generalization Ability Based on Learning Models and Learning Motivation

Learning motivation	PBL	IBL
High	88,48	79,05
Moderate	75,76	81,41
Low	69,10	76,04

Students' mathematical generalization skills vary depending on the learning models and degree of learning motivation, as seen in Table 3. At the high level of learning motivation, the average mathematics generalization ability of students taught using the Problem Based Learning (PBL) model was greater than that of students taught using the Inquiry Based Learning (IBL) model. However, at both the moderate and low levels of learning motivation, the average mathematical generalization ability of students taught in IBL classrooms was higher than that of the PBL group. According to the aforementioned results, learning models have varying effects on students' mathematical generalization skills at various learning motivation levels. The difference in average values at each level of learning motivation supports the results of the two-way ANOVA study, which found a significant interaction effect of learning models and learning desire on students' mathematical generalization ability ($\text{Sig} = 0.009 < 0.05$). Therefore, the impact of learning models on the growth of mathematical generalization abilities depends on both the learning models and the students' desire to learn. The interaction plot between learning models and learning motivation is shown in Figure 1.

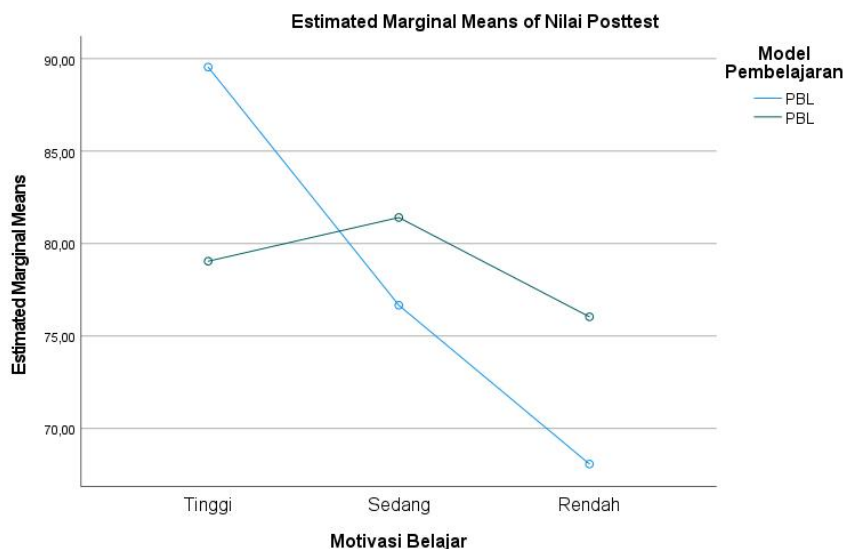


Figure 1. Interaction Plot of Learning Models and Learning Motivation on Students' Mathematical Generalization Ability

Based on Figure 1, it is evident that there is no parallelism between the mean lines of mathematical generalization ability of problem-based learning (PBL) and inquiry-based learning (IBL) models; instead, both lines intersect each other. It means that there exists disordinal interaction between learning models and learning motivation with respect to mathematical generalization abilities of students. If the students have high learning motivation, the mean line of mathematical generalization ability of PBL is larger than that of IBL. Conversely, if the students have moderate and low learning motivations, the mean line of mathematical generalization ability of IBL is larger than that of PBL. The findings demonstrate that the effectiveness of learning model for developing the mathematical generalization ability depends on the level of students' learning motivation. Therefore, the effect of learning model on mathematical generalization ability is dependent on the personal characteristics of students' learning motivation.

Differential Effectiveness of PBL and IBL Across Motivation Levels

The results from the average post-test scores show that highly motivated learners that undertook PBL scored better on mathematical generalization skills than the IBL learners. The pattern implies that highly motivated learners can effectively utilize the challenging nature of problem-based learning settings. PBL entails proactive analysis of problems, searching for pertinent information, developing problem-solving strategies and constructing knowledge independently. Highly motivated learners tend to have much enthusiasm in undertaking these activities, possess higher persistence in task completion and make efforts in looking for other possible solutions. Such qualities greatly help in meeting the aspects of mathematical generalization like recognizing patterns, determining mathematical relationships and making

general conclusions. This observation is consistent with findings by Pujiastuti and Haryadi (2023).

On the other hand, in the case of moderately and poorly motivated learners, IBL produced higher results than PBL. The reason behind such results is associated with the well-defined structure of IBL which helps learners go through different stages including observation, posing problems, forming hypotheses, collecting data, testing hypotheses, and drawing conclusions. The guided process acts as an important scaffolding tool (Reeve & Cheon, 2021), which learners need to construct conceptual knowledge and find relationships between mathematical concepts. Consequently, students with moderate or low motivation receive appropriate structural support, allowing them to optimize their generalization skills without experiencing cognitive overload, which often occurs in the more open-ended and self directed environments of PBL.

Results of Scheffe’s Post Hoc Test

The results of the two-way ANOVA showed that students' mathematics generalization skills are significantly impacted by the learning incentive factor. To find out whether pairs of learning motivation groups showed significant differences, Scheffe's post-hoc test was used. Table 4 displays the outcomes of Scheffe's post-hoc test.

Table 4. Results of Scheffe's Post-Hoc Test for Mathematical Generalization Ability Based on Learning Motivation

(I) Motivation Category	(J) Motivation Category	Mean Difference (I – J)	Sig.	Interpretation
High	Moderate	4,4306	0,312	Not significantly different
High	Low	12,5550	< 0,001	Significant difference
Moderate	Low	8,1244	0,032	Significant difference

The mathematical generalization skills of the two student groups—high learning motivation and moderate learning motivation—do not significantly differ, according to the results of Scheffe's post hoc test in Table 4 (Sig.= 0.088>0.05). However, a significant difference was found between the two student groups: moderate learning motivation and low learning motivation (Sig.= 0.033<0.05) and high learning motivation and low learning motivation (Sig.<0.001). This means that the students with low learning motivation have less mathematical generalization abilities than the students with high and moderate learning motivation.

This finding is consistent with Wang & Guo (2026), who demonstrated that learning motivation positively influences students' persistence in completing mathematical tasks that require higher-order thinking skills. The more motivated students are, the more likely they are

to persist in the process of recognizing patterns and formulating generalizations. This finding suggests that learning motivation is a crucial intrinsic element that helps in the formation of the generalization ability in mathematics. Students with high learning motivation usually show high involvement in instructional activities and become more engaged in the process of pattern recognition, discovery of mathematical relations, formulation of general rules and application of generalizations in various contexts. On the contrary, students with low learning motivation usually face problems with optimal performance of these activities.

The findings of this study are consistent with Ryan and Deci (2020), who pointed out that high learning motivation increases students' engagement in the learning process which leads to improved academic achievements. This conclusion can also be drawn from Lo et al. (2022), who demonstrated that high learning motivation is associated with high levels of persistence, engagement, and participation in the learning process. These conditions allow students to become more engaged in the process of conceptual understanding and higher order mathematical thinking, including mathematical generalization ability.

The Crucial Role of Learning Motivation in Mathematical Generalization

Learning motivation exerts a considerable effect on mathematical generalization, thus pointing to the essential influence of affective factors in mathematics education. Mathematical generalization demands not only cognitive conceptual understanding but also specific affective characteristics, namely persistence, attentiveness, curiosity, and readiness to explore different problem solving strategies. High-motivated students are more active during lessons, more self-assured in expressing their thoughts, and more perseverant when they face challenges while seeking patterns. Hence, they are able to ensure deeper and meaningful learning.

This empirical result is well reflected by Self-Determination Theory (Ryan & Deci, 2020), according to which the presence of intrinsic motivation leads to higher engagement in studying processes and improved quality of cognitive processing. As far as mathematics are concerned, highly motivated students demonstrate high levels of cognitive engagement which makes it possible for them to discover complex patterns, connections between abstract concepts and construct rules. In case of low-motivated students, their interest is limited to following procedures only without any conceptual engagement.

This conclusion is also supported by the findings of foreign research studies (Lo et al., 2022; Xiao et al., 2021; Hidayatullah et al., 2024) according to which the positive correlation was established between learning motivation and student engagement, persistence in the course of academic activities and level of mathematics performance. High-motivated students

sustain longer periods of attentiveness, actively engage in academic discussions and are perseverant in performing challenging tasks which are essential components of pattern observation, conjecture formulation and generalized rules construction.

Implications for Mathematics Instruction

Moreover, the result from Scheffe's post-hoc test shows that students with low learning motivation scored significantly lower in mathematical generalization as compared to students with high and moderate levels of learning motivation. Hence, this finding clearly highlights the role of low learning motivation as a major hindrance to the development of higher order mathematical thinking. Thus, interventions in teaching methods not only depend on choosing innovative models of learning but also on strategies that increase the level of students' motivation. This can be done by providing constructive feedback, local context problems and building a motivating classroom environment (Fianingrum et al., 2023).

On the basis of Vygotsky's theory of social constructivism, the ability of a learner to construct his/her own knowledge is influenced by situational factors present in the learning environment. Highly motivated students perform better in an open autonomous exploration approach such as the PBL. On the other hand, students with moderate and low learning motivation need guided support to achieve effective comprehension. It further strengthens the idea that there is no instructional model which can be superior to others irrespective of the type of students. In other words, the role of learning motivation is very significant as a moderating variable between the instructional model and mathematical performance of the students (Rafiq et al., 2023; Arifin et al., 2025).

In practice, it is important for teachers to adopt differentiated teaching approaches considering the motivational characteristics of the learners. Low motivated students need to be taught using guided tasks, contextualized problems, and scaffolding to help them understand mathematical relations. High motivated students should be given challenging tasks such as high level reasoning problems and exploratory tasks.

CONCLUSION

The current study has demonstrated that the two learning models—the Problem-Based Learning (PBL) model and the Inquiry-Based Learning (IBL) model—have no discernible effects on students' mathematics generalization skills. However, it is evident that learning motivation is crucial to the development of mathematical generalization skills, with high learning motivation producing better outcomes than medium and low learning motivation.

SUGGESTIONS

According to the research findings, mathematics teachers should take into account the motivational attributes of their students before using any model of teaching in the classroom. The PBL teaching model is recommended to be used by mathematics teachers when dealing with highly motivated learners, while the IBL model would work well for moderately and poorly motivated students. Future research is needed to examine some other important factors influencing mathematical generalization ability of students as well as involve a larger and more diverse sample in order to generalize research findings.

REFERENCES

- Alzahrani, K. S., & Alfadhli, E. (2023). The effectiveness of Driver's model in teaching mathematics for developing intermediate school students' mathematical generalization and conceptual understanding. *European Journal of Mathematics and Science Education*, 4(2), 75–88.
- Arifin, Z., Sukarmin, S., Saputro, S., & Kamari, A. (2025). The Effect of Inquiry-based Learning on Students Critical Thinking Skills in Science Education: A Systematic Review and Meta- Analysis, *EURASIA Journal of Mathematics, Science and Technology Education*, 2025, 21(3), em2592
- Bron, J. F., & Prudente, M. S. (2024). Examining the effect of problem-based learning approach on learners' mathematical creativity: A meta-analysis. *International Journal of Research in Education and Science*, 10(3), 653–668. <https://doi.org/10.46328/ijres.3456>
- Dani, S., Pujiastuti, H., & Sudiana, R. (2017). Realistic mathematics education approach to improve students' mathematical generalization ability [in Indonesian]. *JPPM: Jurnal Penelitian Dan Pembelajaran Matematika*, 10(2), 182–193.
- Devi, M. S. A., & Amir, M. F. (2021). Analysis of elementary school students' conceptual and procedural errors in generalizing number patterns [in Indonesian]. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 10(3), 1336–1350. <https://doi.org/10.24127/ajpm.v10i3.3713>
- Doz, D., Žakelj, A. & Cotič, M. (2025). Inquiry-based learning in Grade 9 mathematics: assessing outcomes across Gagné's taxonomy. *Educ Stud Math*, 269–298 <https://doi.org/10.1007/s10649-025-10417-w>
- Ellis, A. B., Lockwood, E., & Ozaltun- Celik, A. (2022). Empirical re-conceptualization: From empirical generalizations to insight and understanding. *Journal of Mathematical Behavior*, 65, Article 100928. <https://doi.org/10.1016/j.jmathb.2021.100928>
- Ellis, A. B., Dogan, M. F., & Knuth, E. J. (2022). Historical overview of studies on generalisation in mathematics education. *International Electronic Journal of Elementary Education*, 14(1), 1–18.
- García Fajardo, V. A., Burbano González, D. C., & Urbano-Leon, C. L. (2025). Mapping the transition from arithmetic to algebra: a bibliometric and structural review (2003–2023). *Front. Educ.* 10:1609194. doi: 10.3389/educ.2025.1609194

- Fianingrum, F., Pujiastuti, H., & Fathurrohman, M. (2023). Disposisi Berpikir Kritis Matematis dalam Pembelajaran Matematika: Systematic Literature Review. *JIIP: Jurnal Ilmiah Ilmu Pendidikan*, 6(1), 543–548.
- Hadi, I., Sariyasa, S., & Suharta, I. G. P. (2025). Meta- Analysis: The Effectiveness of the Realistic Problem- Based Learning Model on Elementary Students Critical Thinking and Mathematical Problem-Solving Skills. *Interational Journal of Education, Management, and Technology*, 3 (3), 965 – 983
- Hidayatullah, A., Abidin, R., Muqit, A. (2024). Motivation and Behavioral Engagement: The Mediating Role of Mathematics Self-Efficacy in Primary Education. *Journal on Efficiency and Responsibility in Education and Science*, 17 (3), 237-246
- Howard, J. L., Gagné, M., & Bureau, J. S. (2017). Testing a continuum structure of self-determined motivation: A meta-analysis. *Psychological Bulletin*, 143(12), 1346–1377.
- Lo, K. W. K., Ngai, G., Chan, S. C. F., & Kwan, K. P. (2022). How students' motivation and learning experience affect their service-learning outcomes: A structural equation modeling analysis. *Frontiers in Psychology*, 13, Article 825902. <https://doi.org/10.3389/fpsyg.2022.825902>
- Maula, N. F., Suryadi, D., & Jupri, A. (2024). Learning obstacles in the generalization process: In case number pattern topic. *Research and Development in Education (RaDEn)*, 4(2), 964–976. <https://doi.org/10.22219/raden.v4i2.36342>
- Muslimah, A., Pujiastuti, H., Mustafa, A. N., & Novaliyosi. (2025). Analysis of junior high school students' mathematical generalization abilities in Cilegon City [in Indonesian]. *JMPM: Jurnal Media Pendidikan Matematika*, 13(2), 797–807.
- Mustafa, A. N. (2023). Transformative approaches and challenges in 21st century mathematics education: A comprehensive review. *World Journal of Advanced Research and Reviews*, 20(3), 444–457. <https://doi.org/10.30574/wjarr.2023.20.3.2508>
- Mustafa, A. N. (2024). Mathematics anxiety: Causes, consequences, and coping strategies. *International Journal of Advanced Research*, 12(11), 1268–1276. <https://doi.org/10.21474/IJAR01/19941>
- Nagy, R. P., Martin, A. J., & Collie, R. J. (2022). Disentangling motivation and engagement. *Frontiers in Psychology*, 13, Article 1045717. <https://doi.org/10.3389/fpsyg.2022.1045717>
- Naraswari, T. P., Kusmayadi, T. A., & Fitriana, L. (2023). Students' mathematical generalization in solving numeracy problems. *Jurnal Penelitian Dan Pengembangan Pendidikan*, 7(3), 533–542. <https://doi.org/10.23887/jppp.v7i3.66888>
- Pujiastuti, H., & Haryadi, R. (2023). Higher-order thinking skills profile of Islamic boarding school students on geometry through the STEM-based video approach. *International Journal of STEM Education for Sustainability*, 3(1), 156–174. <https://doi.org/10.52889/ijses.v3i1.135>

- Pujiastuti, H., & Haryadi, R. (2023b). Hybrid learning impact with augmented reality to improve higher order thinking skills of students. *International Journal of Advanced and Applied Sciences*, 10(12), 7–18. <https://doi.org/10.21833/ijaas.2023.12.002>
- Radford, L. (2021). *The theory of objectification: A Vygotskian perspective on knowing and becoming in mathematics teaching and learning*. Brill. <https://doi.org/10.1163/9789004447055>
- Rafiq, A. A., Triyono, M. B., & Djatmiko, I. W. (2022). The Integration of Inquiry and Problem-Based Learning and Its Impact on Increasing the Vocational Student Involvement. *International Journal of Instruction*, 16 (1), 659-684
- Reeve, J., & Cheon, S. H. (2021). Autonomy-supportive teaching: Its malleability, benefits, and potential to improve educational practice. *Educational Psychologist*, 56(1), 54–77. <https://doi.org/10.1080/00461520.2020.1862657>
- Ryan, R. M., & Deci, E. L. (2020). Intrinsic and extrinsic motivation from a self-determination theory perspective: Definitions, theory, practices, and future directions. *Contemporary Educational Psychology*, 61, Article 101860. <https://doi.org/10.1016/j.cedpsych.2020.101860>
- Saputri, P. T. I., & Purwasih, R. (2024). Pengaruh model *Problem Based Learning* terhadap peningkatan kemampuan berpikir kritis matematis siswa SMP. *JPMI (Jurnal Pembelajaran Matematika Inovatif)*, 7(6), 1045–1054. <https://doi.org/10.22460/jpmi.v7i6.25469>
- Schukajlow, S., Rakoczy, K., & Pekrun, R. (2023). Emotions and motivation in mathematics education: Where we are today and where we need to go. *ZDM–Mathematics Education*, 55(2), 249–267. <https://doi.org/10.1007/s11858-022-01463-2>
- Sen, C., Sonay AY, Z., & Guler, G. (2021). The Effectiveness of Inquiry-based Learning on Middle School Students' Mathematics Reasoning Skill. *Athens Journal of Education*, 8 (4), 417-440
- Simanjuntak, D. A., Makmuri, M., & Rahayu, W. (2021). Problem-based learning: Its effect on higher-order mathematical thinking skills in terms of students' initial abilities and mathematical beliefs. *Al-Jabar: Jurnal Pendidikan Matematika*, 12(2), 457–466. <https://doi.org/10.24042/ajpm.v12i2.10137>
- Stephens, M., Day, L., & Horne, M. (2021). An empirically based practical learning progression for generalisation, an essential element of algebraic reasoning. *Australian Journal of Education*, 65(3), 313–328.
- Suryowati, E., & Trisanti, L. B. (2022). Analysis of students' errors in generalizing patterns based on generalization taxonomy [in Indonesian]. *EDU-MAT: Jurnal Pendidikan Matematika*, 10(1), 106–116. <https://doi.org/10.20527/edumat.v10i1.11250>
- Wang, J., & Guo, L. (2026). Beyond personality traits: a motivation–self-regulation model of mathematical problem-solving through self-efficacy and mathematical thinking. *Front. Psychol.* 17:1803867. doi: 10.3389/fpsyg.2026.1803867

- Xiao, F., & Sun, L. (2021). Students' Motivation and Affection Profiles and Their Relation to Mathematics Achievement, Persistence, and Behaviors. *Frontiers in psychology*, 11, 533593. <https://doi.org/10.3389/fpsyg.2020.533593>
- Yew, E. H. J., & Goh, K. (2016). Problem-based learning: An overview of its process and impact on learning. *Health Professions Education*, 2(2), 75–79. <https://doi.org/10.1016/j.hpe.2016.01.004>
- Yu, L., & Zin, Z. M . (2023). The critical thinking-oriented adaptations of problem-based learning models: a systematic review. *Front. Educ.* 8:1139987. doi: 10.3389/educ.2023.1139987
- Yuni, Y., & Fisa, L. (2020). Guided discovery learning on mathematical generalization ability of junior high school students [in Indonesian]. *Journal of Instructional Mathematics*, 1(1), 20–30. <https://doi.org/10.37640/jim.v1i1.267>