Journal of Green Learning, e-ISSN 2807-890X
Vol. 3, No. 2, December 2023, pp. 72-82.
DOI: 10.53889/jgl.v3i2.328

Enhancing students' mathematical problem-solving proficiency: A synergistic approach of the bar model and butterfly method

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Article Info

Article history:
Received October 28, 2023
Revised December 5, 2023
Accepted December 25, 2023

Keywords:
Bar model
Butterfly method
Math problem-solving

ABSTRACT

The objective of this study is to enhance students' mathematical problem-solving by using a synergistic approach employing the bar model and butterfly method. The sample for this study was 62 seventh-grade students from Kantho Wittayakarn school in Tha Kantho district, Kalasin province during the first semester of the academic year 2022. The research tools consisted of learning management strategy using the bar model and the butterfly method, test for fractal problem solving, and interviewing form for learning activities using the bar model and the butterfly method. The study consistently demonstrated a noteworthy enhancement in seventh-grade students' mathematical problem-solving, particularly in the after-school sessions. Statistical analysis revealed a significant increase in learning achievement levels, surpassing the performance observed before school, with a statistical significance level of .05. The findings consistently suggest that the utilization of the bar model and butterfly method within a learning cycle model can serve as a reliable and effective alternative for advancing students' comprehension of mathematical concepts. The results underscore the consistent positive impact of this synergistic instructional approach on students' mathematical problem-solving.

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1. INTRODUCTION

Problem-solving is an important skill that one must have. Problem-solving in mathematics helps students experience how to solve daily life problems by applying their mathematical knowledge and skills. Word problem-solving is one of the important components of mathematical problem-solving and incorporates real-life problems and applications (Azizah, Rohani, & Mokhtar, 2010). There exists a research gap in the literature regarding the efficacy of specific visual models in enhancing students' understanding and proficiency in mathematical problem-solving. While studies recognize the significance of word problem-solving, there is a dearth of research that systematically investigates and compares the impact of instructional methods utilizing visual models, such as the Bar model and Butterfly method, on students' problem-solving abilities. It can effectively bridged the gap between abstract mathematical concepts and real-life problem-solving scenarios is crucial for advancing pedagogical practices in mathematics education.

To underpin this study, we rely on the theoretical foundation of constructivism, which posits that learning is an active process involving the construction of knowledge through authentic and meaningful experiences. The integration of real-life problems aligns with the constructivist approach, fostering a deeper understanding of mathematical concepts by placing them in practical contexts. In addition, cognitive load theory provides insights into the limitations of working memory and the importance of presenting information in a way that minimizes cognitive load. It can offer a graphical representation of mathematical relationships, can potentially reduce cognitive load and enhance comprehension.

The Bar model is a pedagogical approach that uses visual representation to help students better understand and solve mathematical word problems (Walker, 2005). The use of bar diagrams or models is a concrete and tangible way to represent mathematical concepts and relationships in a problem, making it easier for students to grasp and organize the information presented. By drawing and labeling bars to represent quantities and relationships in a problem, students can more easily identify the relevant information and determine what operations are required to solve the problem. Yeap (2011) has shown that it is an effective teaching and learning strategy for both arithmetic and algebraic word problems. It has been particularly successful in Singapore, where it is taught to primary school students who may benefit from concrete materials and approaches to problem-solving (Veloo & Lopez, 1994; Huat, 2001). By helping students to visualize the problem and break it down into smaller parts, it also can help to build students' confidence in their ability to solve mathematical problems. By providing a concrete and visual representation of mathematical concepts, it can help to make abstract ideas more accessible and understandable for learners at all levels (Kho, 2009).

The study aimed to develop an intervention that integrates two visual methods, to teach the addition and subtraction of fractions. The researchers aimed to create a teaching approach that is easy for students to understand and apply (Rosli, Han, Capraro, & Capraro, 2013). The Butterfly method is a visual and alternative method for adding and subtracting fractions that can help students develop a mental picture of the algorithm. Meanwhile, the Bar model concept uses bars to represent fractions and can help students visualize and understand the concept of adding and subtracting fractions. Students can benefit from the strengths of each approach and develop a more comprehensive understanding of the topic (Madani et al., 2018).

The use of visual images in teaching mathematical concepts has been found to be effective in helping students to understand and solve mathematical problems. Students can develop a deeper understanding of the underlying concepts and apply them more confidently in problem-solving situations (Thirunavukkarasu & Senthilnathan, 2014). The intervention developed in this study has the potential to be a useful teaching approach for educators seeking to improve their students' understanding of fractions. It has been shown to be effective in previous research, the intervention may provide a powerful tool for enhancing students' math skills and confidence (Sharma, 2010; Cardone, 2015; Miller & Obara, 2017).

The difficulty that students experience in learning fractions is a well-documented problem in mathematics education. While fractions are introduced at an early stage in primary school, many students struggle to understand and apply the complex procedures involved in fraction arithmetic. This can be due to the multifaceted computation of fractions, which can be challenging for many students to grasp. The opaqueness of procedures is another issue that can make fraction arithmetic difficult for students. This means that the steps involved in solving fraction problems may not be clear or transparent to students, making it more challenging for them to understand and apply the relevant procedures (Idris & Narayanan, 2011; Lortie-Forgues & Siegler, 2015). This is crucial to ensure that students have a solid foundation in fraction arithmetic at an early stage. This can help to ensure that they have the necessary skills and knowledge to progress to more advanced math topics in the future. By mastering fraction arithmetic at the root, students can develop a more intuitive understanding of the underlying concepts and procedures involved in fraction arithmetic, making it easier for them to apply these skills in more complex problem-solving situations (Lortie-Forgues & Siegler, 2015).

It is important for teachers not only teach the procedures for solving fraction problems, but also to help students understand the underlying concepts and principles. This can be achieved by using visual aids such as the bar model and relating it to the butterfly method algorithm. In addition, it is important for teachers to create a learning environment that encourages students to actively engage with the material and to make connections between different concepts. By doing so, students can develop a deeper understanding of fractions and their applications, which can help them to better solve problems and succeed in future math courses.

2. METHOD

This study utilized a mixed-methods approach to investigate the new teaching strategies using the bar model in combination with the butterfly method. The phases of the conceptual framework for research are depicted in Figure 1.

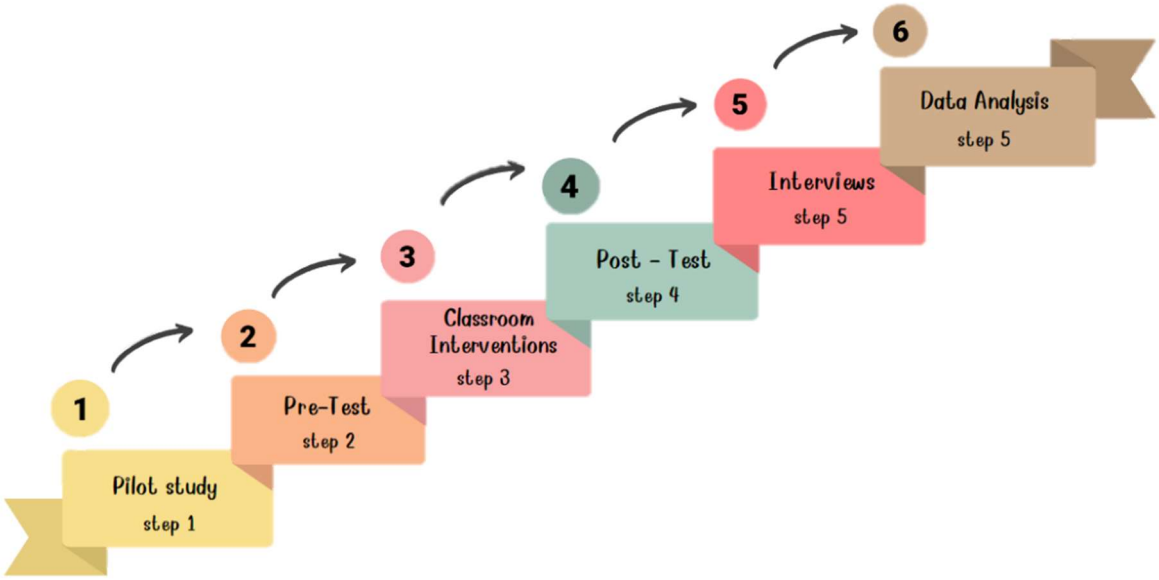


Figure 1 The conceptual framework

This first phase of the study focused on using a quantitative approach to investigate the errors and misconceptions of seventh-grade students in the addition and subtraction of fractions. The research instrument used in this study was adapted from a previous study conducted by Idris & Narayanan (2011). The researchers modified the instrument slightly with different questions while maintaining the same item levels. The pre- and post-tests were used to assess students' prior knowledge and procedures in attempting the questions and to determine if the butterfly method algorithm was effective in solving various types of fractions. Descriptive statistics were used to analyze the test comparisons. In addition, the written analysis from the research instrument was used to identify the errors and misconceptions that students developed in computing fractions that could affect their conceptual understanding.

The interviews were conducted to gain a deeper understanding of the students' experiences with the bar model and the butterfly method. The qualitative data from the interviews were analyzed to identify themes and codes that would help to explain the students' misconceptions and errors. The themes and codes were then used to provide insights into how the intervention impacted the students' ability to add and subtract fractions, as well as their overall mathematical disposition. By examining the students' experiences and perspectives, the researchers aimed to gain a better understanding of how the new teaching strategies were working and how they could be improved in the future.

The study was conducted in the first semester of the academic year 2022, and the sample was taken from 62 seventh-grade students at Kantho Wittayakarn school in Tha Kantho district, Kalasin province. Out of these, 31 students were selected based on the subjects they were taught and their age range of 13 years, reflecting a diverse range of abilities as per their previous examination results.

The data collection methods utilized in this study consisted of several techniques, including a mathematical problem-solving test that was designed by the researcher and administered to 31 seventh-grade students. The test was timed and lasted for one hour, and the

scores were recorded. Additionally, the researcher implemented a learning management approach that incorporated the use of the bar model in conjunction with the butterfly method. The researcher acted as the self-learning manager, and the learning management approach took a total of six hours to complete. Finally, interviews with students were conducted to gather feedback on the learning activities that utilized the bar model and the butterfly method. The learning management plan that incorporated the bar model and the butterfly method was also employed in this study.

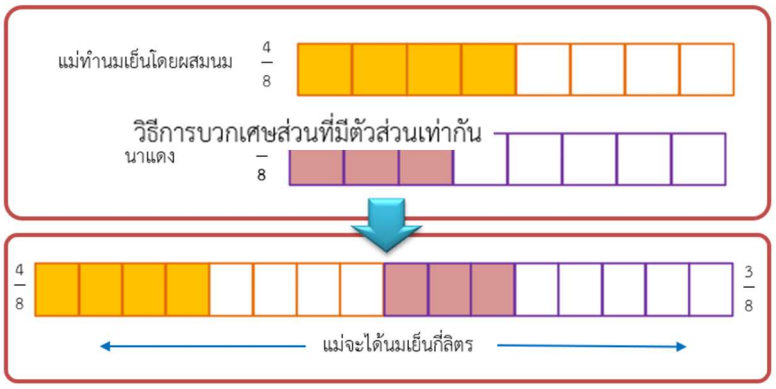
Table 1 Learning plan

Introduction to the lesson (10 minutes)	1. Teachers greet students and inform learning objectives. 2. The teacher reviews the subject's prior knowledge. 3. Teachers give examples of problems. แม่ทำนมเย็นโดยผสมนม $\frac{4}{8}$ ลิตร และน้ำแดง $\frac{3}{8}$ ลิตร แม่จะได้นมเย็นกี่ลิตร
Tutorials (35 minutes)	Teacher explains and demonstrates solving math problems using the Bar model in conjunction with the Butterfly method to find the answer. **Step 1** Understand the problem - Teachers have students observe the problem and then ask questions to analyze and plan for solving the problem. - โจทย์กำหนดมาบ้าง (แม่ทำนมเย็นโดยผสมนม $\frac{4}{8}$ ลิตร และน้ำแดง $\frac{3}{8}$ ลิตร) - โจทย์ให้ถามอะไร (แม่จะได้นมเย็นกี่ลิตร) **Step 2** Plan a solution by drawing a Bar model - The teacher further explains to the student that planning. Solving that problem, students must read the problem to understand it. - Teachers have students plan solutions, which is the process students use to find answers.

2.1 จากโจทย์เขียนเป็น Bar Model ได้อย่างไร
2.2 จากภาพ Bar Model ใช้วิธีการใดในการหาคำตอบ
2.3 จากภาพ Bar Model เขียนเป็นประโยคสัญลักษณ์ได้ว่าอย่างไร
2.4 จากประโยคสัญลักษณ์ได้คำตอบเท่าใด (โดยใช้ Butterfly Method ในการหาคำตอบ)
2.5 สรุปคำตอบได้ว่าอย่างไร

Step 3 Solve the problem by using Butterfly method
- Teachers show how to implement problem solving as examples on the board as follows.

1. How do you write as a Bar model?



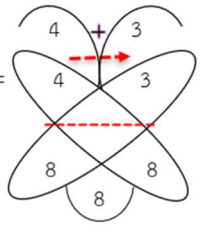
2. Based on the picture, what method does the Bar model use to find the answer?
3. How is the Bar model written in symbolic sentences?

$$\frac{4}{8}+\frac{3}{8}=\square$$

4. From a sentence, how many symbols get answers (using the Butterfly Method to find answers)?

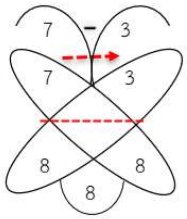
วิธีทำ

$$\frac{4}{8}+\frac{3}{8}=$$



$$=\frac{4+3}{8}$$

แม่จะได้นมเย็น $=\frac{7}{8}$ ลิตร

	5. How to sum up the answer? แม่จะได้นมเย็นทั้งหมด $\frac{7}{8}$ ลิตร **Step 4** Check or look back 1. Teacher describes this procedure as an examination to ensure that the result or answer is correct, complete, logical, reliable, as well as a process for solving problems, which can be used to determine whether the results match. 1.1 ตรวจคำตอบโดย ใช้กระบวนการคิดย้อนกลับในการตรวจคำตอบ ดังนี้ น้ำเย็นทั้งหมด - น้ำแดงที่นำมาผสม = นมที่ผสม $\frac{7}{8} - \frac{3}{8} =$  $= \frac{7-3}{8}$ นมที่ผสมที่แม่นำมาผสม $= \frac{4}{8}$ ลิตร **ดังนั้น** นมที่ผสมที่แม่นำมาผสม เท่ากับ $\frac{4}{8}$ ลิตร
Summary stage (5 minutes)	Teacher has students together to summarize the procedure for solving the problem of adding fractions where the denominator is equal.

3. RESULT AND DISCUSSION

The overall results of the students. It indicates that the students showed progress in the first post-test that was conducted after the intervention is shown in Table 2.

Table 2 The overall pre-test and post-test scores of the seventh-grade students

Test	N	Mean	SD	% passed
Pre	31	10.06	2.74	25.15 %
Post	31	30.16	4.58	75.40 %

The highest score among the 31 participants before the intervention was 15, while the lowest was 5. However, one participant who had a score of 44 in the first post-test experienced a slight decline but was able to score 38, which is an improvement in the second post-test. The lowest score of 20 was achieved by a low-ability participant who showed a slight improvement in the first post-test. In the first post-test, 75.4% of participants passed the test, while in the second post-test, the percentage of participants who passed the test decreased to 25.15%. The passing mark for the test was set at 50%

When considering the test, which contained 4 questions, the first of two questions were easily answered by all 7^{th} grade students. Question 3, which assesses the proper addition of fractions with different denominators, posed a challenge for 40.50% of the students during the pre-test, with incorrect shading of the Bar model. However, after the intervention, this percentage decreased to 14.20%, with minor errors and misunderstandings observed in the students' written analysis. The students were able to connect the shading of the visual bar model to the addition of fractions, while the butterfly method was used to add the two given fractions.

After the intervention, there was a slight improvement in the scores for Question 4, which involves the subtraction of fractions with different denominators. However, some students still showed misconceptions in using the Butterfly Method to subtract fractions. In the pre-test, 51.6% of the students had difficulty subtracting fractions with different denominators, while this percentage decreased to 29% in the post-test. Although the results show an improvement, the percentage of students who still struggle with the subtraction of fractions with different denominators is still relatively high.

The mean score for attempting Question 4 denominators from the pre-test was 45.1%, and the post-test score displayed an increased score of 69.8%. There was a difference of 24.7% between the pre-test and post-test scores. Overall, the students had made improvements after the intervention for attempting Questions 3 and 4. The percentage decrease in incorrectly shading the Bar models and the increase in the percentage mean score after the intervention. Students’ performance in adding and subtracting fractions with different denominators gained students' mathematical problem-solving proficiency has increased consistently. It is suggesting that the utilization of the Bar model can serve as a reliable and effective alternative for advancing students' comprehension of mathematical concepts. The results underscore the consistent positive impact of this synergistic instructional approach on students' mathematical problem-solving proficiency.

In addition, the researcher also conducted interviews with students. It was found that a total of eight students were selected for the interviews. Students were selected based on their post-test percentage mean scores and the occurrences of the errors, which were analyzed from their written responses. Students were classified into three groups according to the specified ranking, such as low performance, average performance, and high performance. The students were interviewed in the mathematics resource room during their break time. The interviews were conducted for approximately 7 to 10 minutes, either individually or in pairs. However, the male students who were interviewed in groups of five lasted approximately 15 to 25 minutes. All interview sessions were audio-recorded and transcribed.

Theme 1: Summary of the students' responses on their views regarding the learning activities using the bar model and the butterfly method.

All students who were interviewed provided positive feedback on using the Butterfly Method for adding fractions, but expressed difficulty with subtracting fractions, particularly those with the same denominators. Many students made careless errors, and some forgot to reduce their answer to the lowest term. Table 3 provides a summary of the students' feedback.

Table 3 Summary of the students' responses on their views regarding the learning activities using the Bar model and the butterfly method

Advantages	Students' opinions
Visual	• Students had an easier time remembering the steps by drawing the butterfly on the fractions
Ease of use	• The butterfly is simple to draw • Need further instruction and practice using the butterfly method to solve various fractional addition and subtraction problems • It is much simpler to utilize the butterfly method to add fractions with different denominators than it is to reduce fractions with the same denominators
Effectiveness	• The Bar models are effectively divided • When applying the butterfly method, helps to check and respond to the offered fractions

The students' lack of confidence in applying the Butterfly Method despite being aware of its benefits was evident in some students, such as S11 and S12. These students expressed their desire to use the method in the future but felt that they needed more guidance and support from their teacher. To build the students' confidence and familiarity with the Butterfly Method, more practice should be provided to them. Wong & Evans (2007) suggest that practicing the routine or procedure can help students recall it more easily, build their confidence, and become more familiar with the algorithm.

It's important for teachers to ensure that students understand the underlying concepts of fractions and not just memorize procedures. The use of Bar models can help students visualize the relationships between fractions and improve their understanding of the concepts. Additionally, practicing with visual manipulatives and giving students ample time to practice on difficult questions, especially with the subtraction of fractions, can further enhance their learning. Teachers should also emphasize common mathematical errors and misconceptions to help students avoid making careless mistakes. Prior to implementing new teaching strategies, it's essential to ensure that students have mastered multiplication tables and the concept of equivalent fractions (Sarwadi & Shahrill, 2014).

It has been suggested by Hunter et al. (2020) that teachers tend to focus on repetitive practice exercises instead of encouraging collaborative discussions that can promote students' comprehension of mathematics. To enhance learning, students should be given opportunities to ask questions, express their opinions, and justify their mathematical reasoning. This type of

active engagement in learning can boost students' creativity and confidence, thus reducing uncertainties and misunderstandings when learning new concepts. As Zakir (2018) stated, being a teacher requires the ability to adapt to changes in their teaching practices.

4. CONCLUSION

In this study, the students' feedback indicated that the use of Bar model during the intervention was beneficial in visually assisting them in splitting rectangular bars based on the given fractions. It enabled students to verify their answers using the butterfly method. They found that shading the bars for adding fractions and reducing them for subtracting fractions was particularly helpful. In addition, the students who did not use the butterfly method during the post-tests found the Bar model helpful in checking their answers when comparing them with the formal method. The majority of the students who applied the butterfly method expressed satisfaction with the new method, finding it easier to remember than cross-multiplying the numerator with the denominator. However, students who lacked confidence or struggled with applying the butterfly method when subtracting proper fractions from whole numbers, subtracting proper fractions with the same denominator, or making careless errors required additional support and practice from the teacher.

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